Dipole-Dipole Interaction of a Non-Linear Pendulum

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Introduction

In the past, the chaotic pendulum has been dealt with theoretically by computer simulation. Our group has constructed a simple experiment that generates the chaotic behavior in real time expected fom the theoretical, computer-generated model. The goal of our research has been both to match a theoretical and experimental pendulum, but also to explore the effect of a double magnet interaction on the nonlinear motion we observed. The net torque equation on a dipole pendulum is:

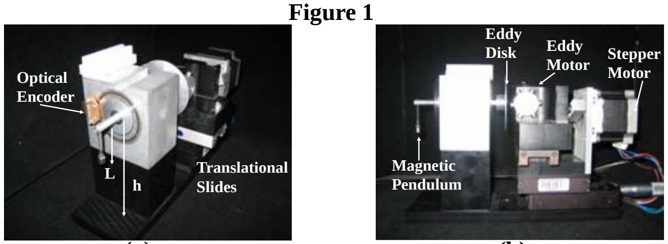
$$Eq(1)\quad I\ddot{\theta} = -mgL\sin(\theta) - b_F\dot{\theta} - \tau_F\left[\frac{\dot{\theta}}{|\dot{\theta}|+\varepsilon}\right] + \tau_D\cos(\omega_D t+\delta) + \tau_{DD}$$

Net Torques = (gravitational) (dynamic and static friction) (sinusoidal driving) (dipole-dipole)

We fit this equation to the experimental data using a fourth-order Runge-Kutta algorithm for the pendulum without dipole and an eighth-order Runge-Kutta algorithm for the dipole-dipole pendulum. We isolated the angular acceleration by scaling all coefficients to the moment of inertia, I. The dipole dipole torque, expanded in Eq (2), is a function of the physical dimensions of the system, as shown in Figure 1(a), and the angular position of the pendulum.

$$Eq(2)\quad \tau_{DD}(\theta) = -\frac{m_1 m_2 \mu_0 \sin\theta}{8\pi(h^2+L^2-2hL\cos\theta)^{7/2}}\left[\begin{matrix}4h^4+37h^2L^2+10L^4-hL(16h^2+61L^2)\cos\theta+ \\ L^2(11h^2+18L^2)\cos(2\theta)-3hL^3\cos(3\theta)\end{matrix}\right]\hat{z}$$

Figure 1



(a) **(b)**

The experimental setup, pictured in Figure 1(b), includes a sinusoidal driving system using an eddy motor with rotating magnets creating eddy currents in an aluminum disk attached to the pendulum axel. The pendulum is driven at an amplitude and frequency such that the resulting motion is chaotic. To create the dipole-dipole system, a magnet was attached to the end of the pendulum and an opposing magnet was fixed underneath. This leads to changes in the potential well which are illustrated to the right.

Reference:
DeSerio R., *Chaotic Pendulum: The Complete Attractor*, Am. J. Phys., Vol. 71, No. 3, March 2003

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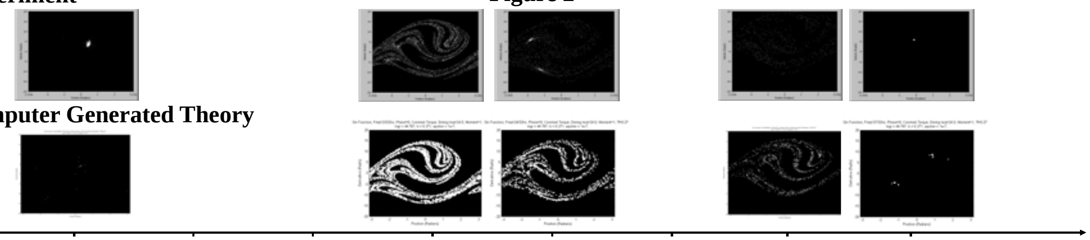
Figure 2

Experiment

Computer Generated Theory



f(Hz) **20/32** **23/32** **24/32** **26/32** **27/32**

Figure 2, above, shows an experimental-theoretical match of bifurcation through driving frequency. A bifurcation tracks the progression of data when all variables but one are held constant. The friction, gravitation, zero dipole, and driving strength were held constant while the driving frequency was changed.

Figure 3, below, shows an experimental-theoretical match of bifurcation through dipole strength and the potential energy curve for each dipole strength. The “pinching” effect of the magnetic dipole interaction is clearly seen. The Poincare section appears to mimic the double well in the potential energy plots.

Figure 3

Experiment

Computer Gen