Constancy Amid Chaos: Invariant Measures in Non-Linear Dynamics

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Introduction:

This project is a continuation of work started in the summer of 2004 with the original chaotic pendulum system. The apparatus has been modified with the addition of a magnetic dipole-dipole interaction as explained on the sister poster to this one. This magnetic interaction changes the potential and causes appreciable qualitative differences in the behavior of the system. This project seeks to quantify those differences and uses the tools of nonlinear time series analysis.

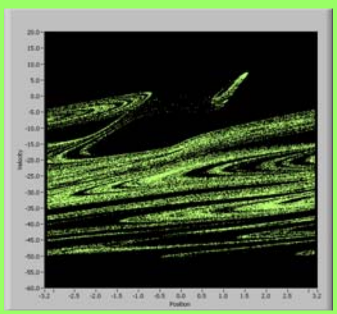
Background:

According to Henry D.I. Abarbanel, chaotic signals lie somewhere between the well studied domain of the predictable, regular, or quasi-periodic which have been the mainstay of signal processing and the totally irregular stochastic signals we call 'noise' which are completely unpredictable.

This necessitates different methods of attack for chaotic signals. With more predictable time series, tools like Fourier analysis may provide insights by looking at the system in terms of frequency; chaos is broadband noise in the frequency domain. Chaos, however, does have structure in phase space. The behavior of the system is governed by strange attractors and quantities like fractal dimension and Lyapunov exponents that describe the attractor provide invariant measures of system.

Experimental Indicators:

One indication that the behavior of the system with the magnetic dipole-dipole interaction was different appeared when we looked at the Poincaré sections, (slices in phase space). We noticed there were 'drips,' groups of points that would become separated from the main structure present in the section. There was also 'folding-over' present indicating a higher dimensional system. This behavior was not observed without the dipole-dipole interaction.



Poincaré Drips

Attractor Reconstruction:

We immediately face a challenge when we seek to more thoroughly analyze the phase space of an experimental system; we don't have it. Unlike in a model where we have access to all the parameters that specify the state of the system, in an experimental system we usually only have access to a time series of one dynamical variable. For example, in our system, we have the angular position of the pendulum as a function of time.

However, embedded in the time series for one variable is information about the others. This can be exploited to produce reconstructions of the phase space for the system using a process known as time-delay attractor reconstruction. Thankfully, although a time-delay reconstructed attractor is different than the actual underlying attractor it has the same invariant characteristics, like Lyapunov exponents. To illustrate the procedure:

Given a time series of one variable:

$$(x_0, x_1, x_2, \ldots x_n)$$

A time delay reconstruction for a given delay time (τ), and embedding dimension (d), would be:

$$\left((x_0, x_{0+\tau}, x_{0+2\tau}, \ldots x_{0+(d-1)\tau}), (x_1, x_{1+\tau}, x_{1+2\tau}, \ldots x_{1+(d-1)\tau}), \ldots\right)$$

The quality of the reconstruction and as a result measures obtained from it are dependant upon the embedding dimension, d, and time delay, tau. There are, however, statistical measures that can indicate good parameter value choices.

False Nearest Neighbors:

Used to decide embedding dimension.

$$\sqrt{\frac{R_{d+1}^2(k) - R_d(k)^2}{R_d(k)^2}} > R_{threshold}$$

Mutual Information:

Used to decide time delay.

$$I(T) = \sum_{s(n),s(n+T)} P(s(n), s(n+T)) \log_2 \left[\frac{P(s(n), s(n+T))}{P(s(n))P(s(n+T))} \right]$$

Current State of Research:

I've written programs in Java to do the following with our experimental data:

- Create Poincaré 'movies' to explore the 'Poincaré drips'
- Create an attractor reconstruction for a given time delay and embedding dimension
- Calculate False Nearest Neighbors as a function of dimension
- Calculate Average Mutual Information as a function of time delay

Future Goals:

- Test my programs against available programs on known data to gain confidence that they are working correctly
- Write a Lyapunov Exponent calculation program from scratch
- Compare Lyapunov Exponents program results against available programs
- Be able to distinguish between the behavior of the chaotic pendulum with and without the magnetic dipole-dipole interaction based on Lyapunov Exponents
- Investigate fractal dimension
- Investigate and attempt to determine Kolmogorov-Sinai Entropy for our system

Selected Sources:

Abarbanel, Henry D.I. Analysis of Observed Chaotic Data NY: Springer, 1996.

R. Hegger, H. Kantz, and T. Schreiber, *Practical implementation of nonlinear time series methods: The TISEAN package*, CHAOS **9**, 413 (1999)
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