Non-Linear Dynamics: Torque and Potential Curves from a Magnetic Dipole-Dipole Interaction

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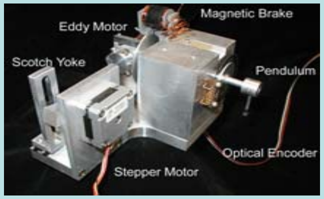
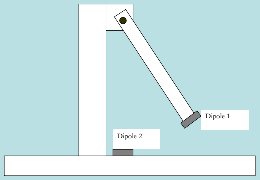
Introduction

The field of non-linear dynamics involves the study of chaotic motion. A simple pendulum can move in a chaotic fashion if torques from non-linear magnetic fields are applied to it. The current experimental setup applies these magnetic fields as eddy currents to a disk attached to the pendulum. This research explores applying another magnetic field, applied directly to the pendulum in this case, with a double-magnet system that creates a dipole-dipole interaction.

The System & The Equation

The current apparatus is pictured below and left, with the eddy currents supplied by the labeled Eddy Motor. The dipole-dipole interaction is created with two additional magnets – one attached to the pendulum, rotating with it, and the other placed on the base of the apparatus, directly under the pivot point. A schematic of this shown below and right.



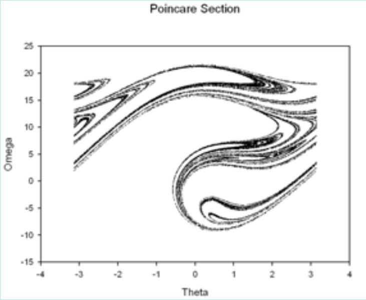


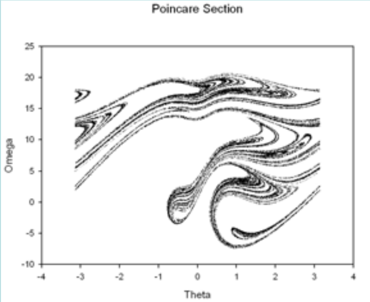
The equation of motion for the system, displayed below and right, includes several torques that affect the pendulum's motion, including frictional and driving. The dipole-dipole interaction produces another torque on the system, which can be derived using magnetic field equations and a careful coordinate system. The equation for the dipole-dipole torque is displayed below and left.

$$\tau_{DD}(\theta) = -\frac{m_1 m_2 \mu_0 \sin\theta}{8\pi(h^2+L^2-2hL\cos\theta)^{7/2}}\left[4h^4+37h^2L^2+10L^4-hL(16h^2+61L^2)\cos\theta+L^2(11h^2+18L^2)\cos(2\theta)-3hL^3\cos(3\theta)\right]\hat{z}$$

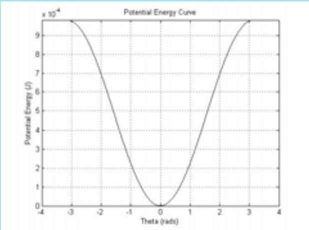
$$I\ddot{\theta} = -mgr\sin(\theta)-[b+\tau_F]\dot{\theta} - \tau_F\left[\frac{\dot{\theta}}{|\dot{\theta}|+\varepsilon}\right]+\tau_D\cos(\omega_D t+\delta)+\tau_{DD}$$

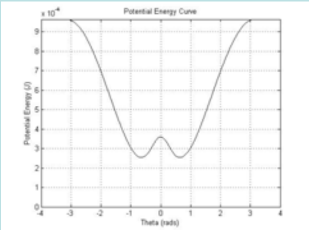
Double-Dip Your Pendulum





The equations in the bottom left corner were solved in MatLab by an embedded Runga-Kutta numerical integration method. The results are graphed in a Poincare section, which is a sampling method of data useful in studying chaos. The graph above and left is the system without the dipole-dipole interaction, while the graph above and right is the system with the interaction. The 'dip' in the center of the graph is possibly caused by the double-well potential created by the dipole-dipole interaction. To see this double-well, the potential energy graph of the system is displayed below, again without the dipole-dipole interaction on the left and with on the right.





Thank You

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