Charge Distribution on 1- and 2- Dimensional Surfaces

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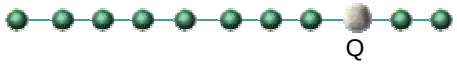
Model Overview

- Question: How does charge arrange itself on a conductor?
- Solution: Model various "metal" shapes, studying how charge spreads and settles on the shapes of one and two dimensions.

- The outcome for 1-D is unknown because:

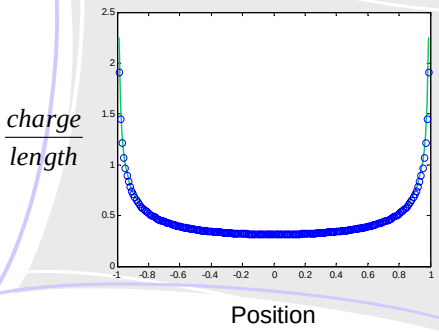
  →The functional description for charge distribution predicts a uniform, even spread along a line.

  →Intuition would suggest an uneven distribution from apparently imbalanced forces. As in the model below, more charge is to the left of "Q" than to the right.

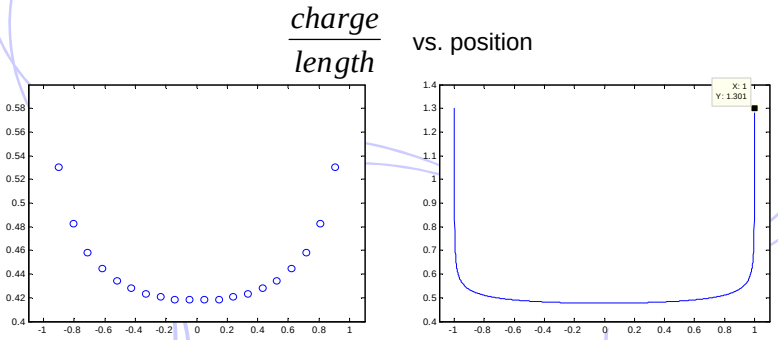


Two Dimensional Model

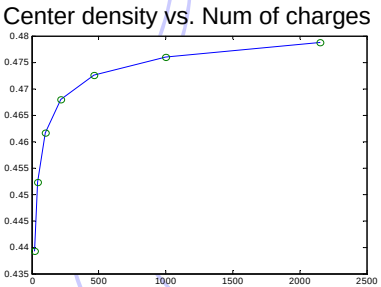
- Similar to the "bead"-ed wire, break a foil into many strips, or wires. Adding more wires simulates a smooth foil.
- The importance of this model is in that the numerical and analytical results match what we intuitively expect, repulsion to the ends of the needle.

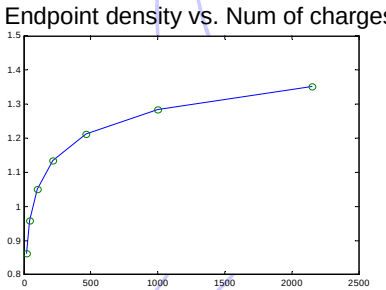


The green line is the functional prediction. The blue dots show the computer-generated approximate values.



Charge density results for 20 point locations and 200 point locations





One Dimension Model

•What is this physically?

A thin needle or short length of wire.

• What math is involved?

Coulomb's Eqn- $F = \frac{kq_1q_2}{(r_{12})^2}\,\vec{r}$

•Program approach: Vary distance in a controlled manner to avoid **r**'s inverse square. Allow charge to vary as needed in the numerator.

•Program results: The controlled variation in distance only allows charges at specific points, as in the picture above.

As the number of locations for charge increases, it simulates a real metal needle in which there is no position restriction.

The evening of charge density is obvious between the graphs of 20 and 200 charges.

However, the plots still show that density spikes at the needle's tips.

An Interesting Conclusion

• The final tests measure how density changes for large N. How does this change as the number of charges grows?

•The center density evens to the analytically predicted value.

•The tip density continues growing for all finite N. In that sense, intuition is justified. However, the overall effect of the tip density shrinks as N grows, which confirms the analytical result.