Non-Linear Dynamics: The Theoretical Model

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Equation of Motion

$$I\ddot{\theta} = -mgr\sin(\theta) - [b+\tau_F]\dot{\theta} - \tau_F\left[\frac{\dot{\theta}}{|\dot{\theta}|+\varepsilon}\right] + \tau_D\cos(\omega_D t+\delta)$$

- Describes the behavior of our system
- Derived from Newton's Second Law

The Basics

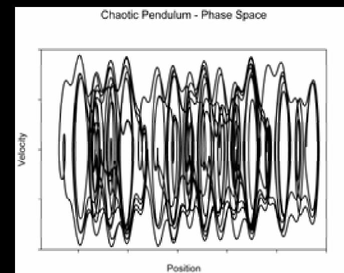
The goal of a theoretical model is to accurately recreate physical phenomena inside of a computer; in this project, the phenomenon is a chaotic pendulum. From basic principles, such as Newton's Second Law, one can derive an equation of motion that generally describes this pendulum (displayed above). Then, this equation is applied to a specific apparatus to make the experimental motion of the pendulum a reality inside of a computer.

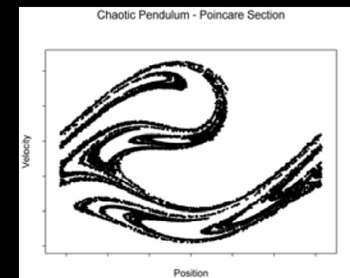
Input Parameters

$b, \tau_F, \tau_D, \omega_D$

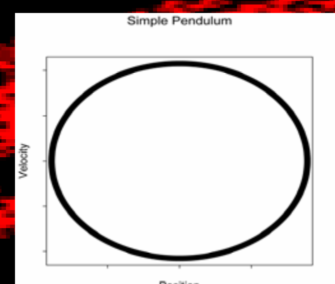
- Experimentally determined
- Unique to each apparatus

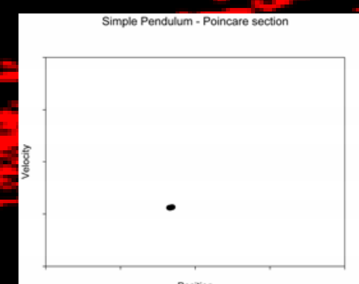
Chaotic





Periodic





Phase Space

These two phase space graphs display the data produced by the MatLab program. Periodic behavior (below) is simple and repeatable, making the graph a smooth ellipse. Chaotic behavior (above) is defined by its unpredictable and intricate nature, making the graph a complex hash.

Poincare Sections

A useful method to observe chaotic behavior is to create Poincare sections, which are made by sampling the position and velocity data from the Phase Space graphs at the driving frequency.

The periodic Poincare section (below) is a single dot. This is analogous to shining a strobe light on a Grandfather clock's pendulum. If the light strobes every two seconds, you will see the pendulum at exactly the same position which, if graphed, would produce a single dot.

The chaotic Poincare section (above) appears to have some organization. The hash on the 'Chaotic Pendulum – Phase Space' graph (above, left) has become smooth curves. This is the magic of chaos: a situation that is unpredictable has a subtle and amazing order.

The Three Stooges of Physics

Zach Simmons, Matthew Jungwirth & Erik Johnson.
The collaborated talents of these three men researched Non-Linear Dynamics during the summer of 2004.

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