Non-Linear Dynamics: Experimental Chaos

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Recipe for Chaos

The word chaos is often used to describe non-linear motion. Our project focused specifically on the chaotic motion of a pendulum. There are two initial conditions for chaos to occur. They are:

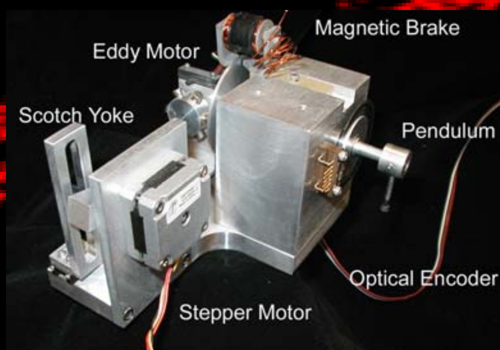
1. Three independent variables
2. A non-linear component of motion

Our system satisfies these requirements using some creative methods of damping and driving the motion of the pendulum. The independent variables in the system come from the friction inherent in the bearings, the magnetic damping from the coil over the aluminum disc and the driving torque provided by the spinning magnets in the back. Our non-linear component comes from the sinusoidal drive.

$$I\ddot{\theta} = -mgr\sin(\theta) - b\dot{\theta} - \tau_F\left[\frac{\dot{\theta}}{|\dot{\theta}|+\varepsilon}\right] + \tau_D\cos(\omega_D t + \delta)$$

Non-Linear Driving Term
Magnetic Damping
Frictional Torque
Driving Torque
Driving Frequency

The System



System Constants

Our system relies heavily on very specific values for the constants. These constants include the moment of inertia, the mass and length of the pendulum, the damping caused by the electromagnet and the driving torque from the spinning magnets. In order to understand our system better and to properly model it mathematically we need values for the constants. To find the values of the constants we used various methods.

The mass and length of the pendulum were easy to find using a scale and a caliper.

To find the driving torque and the magnetic damping we had to do more data gathering and calculation. Using the same programs utilized in experimental data gathering we took controlled data with the pendulum on its side (to negate gravity). This was done for both the magnetic damping and the driving torque. We also did this initially for frictions but realized later that the frictional torque is not constant. The friction depends on the velocity of the pendulum and is therefore dynamic.

Driving our System

Our system is driven by a DC motor with some magnets on it. These magnets produce an eddy current in the aluminum disc and causes it to rotate. This rotation moves the pendulum. However this driving must be non-linear to satisfy the requirements for chaos. To introduce a non-linear factor we move the spinning magnets back and forth horizontally across the disc. This method of driving is sinusoidal and therefore non-linear.

In order to drive the system in this manner we use a stepper motor. The stepper motor is controlled by a circuit designed by Mike Rodning and myself. In order to have sinusoidal driving the stepper motor has to run smoothly. Our driving circuit takes advantage of the stepper motors ability to half step and reduce the amount of noise entered into the system.

The circuit designed to drive the stepper motor must begin with some input signal. This signal a multiple of the frequency at which we want the stepper motor to run (400 times to be precise). Coincidentally in order to take data via the computer we need a sampling frequency to tell the computer when to record data points. For convenience sake Zach created a circuit that takes one signal and splits it into two signals that serve as the sampling frequency and the driving input. Ultimately the frequency of the stepper motors motion is 1000 times less than the sampling frequency. This is convenient for taking Poincare plots.

Gathering Data

To show our system is capable of being chaotic we had to create a way to gather data from the system and analyze it. This is done through and optical encoder and a series of computer programs.

The optical encoder reads the position of the pendulum using a marked disc. The encoder reads the disc and sends a signal out that we wired directly to the DAQ board which sends the signal to the computer. Using a program called LabVIEW Zach was able to create a program that reads the signals from the encoder and writes them to a spreadsheet file along with the time they were recorded.

The resulting spreadsheet file contains the position of the pendulum at a given time. The average file has about 500,000 different positions and their corresponding times. This spreadsheet file can be analyzed using a program written by Matt in the JAVA language. The JAVA program differentiates the position data to get a velocity which is essential for looking at chaos graphically.