Second-order Phase Transitions

Second-order phase transitions occur when a material’s temperature falls below a certain critical temperature, $T_c$, forcing it to change from a normal state to a superconducting state. The transition causes the material’s electrical resistance to drop to zero allowing perfect electrical conduction, the hallmark of superconductivity. Second-order phase transitions are characterized by a sharp rise in specific heat, $C/T$, without an accompanying rise in temperature, $T$. The graph below displays the specific heat versus temperature for the material studied, PrOs$_4$Sb$_{12}$.

Notice that PrOs$_4$Sb$_{12}$ displays two second-order phase transitions, which is an anomaly in nature. This project theoretically studies the cause of the first of these transitions, shown in the box on the graph above.

The System

The macroscopic wave function that describes the system was derived from the Ginzburg-Landau free energy expansion. This equation, shown below, includes terms for a constant, externally applied magnetic field to observe the material in a setting much closer to laboratory conditions.

$$F = \alpha \sum_i \left[ \left( \frac{\mu}{\mu_0} \right) \left( \frac{\mu B_i}{m_i} \right) + \chi \left( \frac{\mu B_i}{m_i} \right) \right] + \kappa \left( \frac{\mu B_i}{m_i} \right)^2 + \frac{\kappa}{2} \left( \frac{\mu B_i}{m_i} \right)^3$$

where

$$\mu = \frac{P \cdot c}{B}$$

$B = \text{curl} \, A$

Results

The graph on the near right shows the ‘bend’ result equal to the spherical result at high fields, while the graph on the far right shows the ‘bend’ test result equal to the cubic result at low fields, just as predicted. Analyzing the ‘bend’ result will reveal the physical properties of the transition, the details of which will be compiled in a later project.

The ‘Bend’ Test

To determine the properties of the transition, a method analogous to a stress test on a structure is used. PrOs$_4$Sb$_{12}$ under a weak field allows coupling of spin and orbit for the electrons, making it appear to have cubic symmetry. However at high fields, spin and orbit are decoupled, appearing as spherical symmetry. Eigenvalues are found by a MatLab program solving the macroscopic wave function at each point in an ever-increasing magnetic field, forcing the system from cubic to spherical symmetry (displayed left). Hence, the system is ‘bent’ by the increased field until it ‘breaks,’ thus revealing its secrets.

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